## Experiment \#2: Determining Sugar Content of a Drink

## Objective

How much sugar is there in your drink?
In this experiment, you will measure the amount of sugar dissolved in a soft drink by using two different methods. The first method is density measurement. And the second method is the index of refraction.

## Introduction

## Density

The density of a substance, is the mass per one unit volume of that substance.

$$
\begin{align*}
& \text { Density }=\operatorname{mass}(\text { gram }) / \text { volume }\left(\mathrm{cm}^{3}\right) \\
& \qquad \rho=\frac{m}{V} \tag{Equation1}
\end{align*}
$$

We can determine the density of a substance by measuring the volume and mass.
We can measure the volume of our solutions with a measuring cylinder, a beaker, a falcon tube or a pipette depending on the capacity and the precision of the given glassware. The precision in each grassware is the half of its smallest scale.
We can measure the mass of the solution with a laboratory scale. The precision of the scale is 0.01 g . Therefore the error in our measurements is $\pm 0.01$.

## Index of Refraction

You may have seen an odd "bending" effect when you put a chopstick in a glass of water: the chopstick appears to bend at the boundary between the air and the water.


Figure 1. This image illustrates refraction by liquids.
The chopstick in Figure 1, above, seem to bend because of a phenomenon called refraction, the bending of a wave as it passes from one material into another. Waves travel at different speeds through different mediums (such as air and water), and this speed difference makes the wave refract when it passes from one material into another. As light waves travel from the
water into the air, the wave refracts, which makes the chopstick in Figure 1, above, look bent or broken.
The denser the medium, the slower the wave travels through the medium. The speed at which a light wave travels through a medium is quantified in the index of refraction, $\mathbf{n}$, of that medium. It represents the ratio between the speed of light in a vacuum and the speed of light in the medium of interest. For example, the index of refraction of air is 1.00028 , so light travels 1.00028 times faster in vacuum than it does in air.
The amount of refraction, or how much a light wave bends when it travels from one medium to another, is related to the indices of refraction by a mathematical formula called Snell's Law. We will use Figure 2, below, to understand Snell's Law. Figure 2 shows a ray of light passing from water to air. Snell's Law uses the angle of a light ray entering a material, called the angle of incidence, $\boldsymbol{\theta}_{\mathbf{1}}$, and the indices of refraction of the two materials (air and water, in this case) to calculate how much the light ray will refract as it passes from one material into the other. "How much the light refracts" is expressed mathematically as an angle called the angle of refraction, $\boldsymbol{\theta}_{\mathbf{2}}$. Note that both the angle of incidence and the angle of refraction are measured from a line perpendicular to the surface the light interacts with. This line is called the surface normal, or simply, the normal (dashed gray line in Figure 2).

(a)

(b)

Figure 2. Illustration of Snell's Law.

The relationship between incident and refracted rays are visualized in figure 2(a). When the angle of incidence increases, the angle of refraction also increases, figure 2(b).
In equation form, Snell's Law looks like this:
Index of refraction of material $1 \times \sin ($ angle of incidence)
=
Index of refraction of material $2 \times \sin ($ angle of refraction)
(Equation 2)

$$
n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}
$$

- $\mathbf{n}_{\mathbf{1}}=$ index of refraction of material 1 (no units, since it is a ratio)
- $\boldsymbol{\theta}_{\mathbf{1}}=$ angle of incidence (degrees or radians)
- $\mathbf{n}_{\mathbf{2}}=$ index of refraction of material 2 (no units, since it is a ratio)
- $\boldsymbol{\theta}_{\mathbf{2}}=$ angle of refraction (degrees or radians)

The index of refraction for air, $\mathrm{n}_{2}=1$.
And Snells law equation becomes:

$$
\mathrm{n}_{1} \sin \theta_{1}=1 \sin \theta_{2}
$$

Therefore the index of refraction for the sugar solution is:

$$
n_{1}=\sin \theta_{2} / \sin \theta_{1}
$$

Wear goggles, gloves and labcoats in the lab at all times.

## Laser Safety:

*Never shine a laser pointer at anyone.
*Do not point a laser pointer at mirror-like surfaces. A reflected beam can act like a direct beam on the eye.

## Equipment for Part A: Density Method

- Sugar solutions
- Soft drinks (A, B)
- Falcon tube
- Laboratory scale
- Beaker


## Equipment for Part B: Index of Refraction Method

- Sugar solutions
- Soft drinks (A, B)
- Optics bench
- Ray table
- Hollow semicircular block
- Laser source
- Falcon tube
- Beaker
- Syringe


## Part A: Determining the content of sugar with density method.

1- The solutions with concentrations of $10 \%, 20 \%$ and $30 \%$, shown in Table 1 are prepared earlier.

Table 1
(Amounts of Sugar and Water for Standard Sugar Solutions of 100 ml .)

| Sugar concentration <br> $(\%)$ | Amount of Sugar <br> $(\mathrm{g})$ | Amount of Water <br> $(\mathrm{mL})$ |
| :--- | :--- | :--- |
| 10 | 10 | 90 |
| 20 | 20 | 80 |
| 30 | 30 | 70 |

2- Weight an empty beaker on the laboratory scale. Record its mass in Table 2.
3- Measure 50 ml of $10 \%$ solution with a falcon tube.
4- Pour the solution in the beaker and measure the total mass of the beaker plus the solution and record the result in table 2.
5- Repeat the procedure for $20 \%$ and $30 \%$ solutions and record your results in table 2.

Table 2

| Concentration of <br> solution <br> $(\%)$ | Volume of <br> solution (ml) | Mass of <br> beaker $(\mathrm{g})$ | Mass of beaker + <br> solution $(\mathrm{g})$ | Mass of <br> solution $(\mathrm{m})$ | Density of <br> solution $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 10 | 50 |  |  |  |  |
| 20 | 50 |  |  |  |  |
| 30 | 50 |  |  |  |  |

Table 3

| Soft Drink | Volume of <br> solution (ml) | Mass of <br> beaker (g) | Mass of beaker + <br> soft drink $(\mathrm{g})$ | Mass of soft <br> drink (m) | Density of soft <br> drink $\left(\mathbf{g} / \mathrm{cm}^{3}\right)$ |
| :--- | :--- | :---: | :---: | :---: | :---: |
| A | 50 |  |  |  |  |
| B | 50 |  |  |  |  |

## Data Analysis for Part A

1- Calculate the mass of solution and record your results in table 2.
2- With the mass and volume of the solution is known, using equation 1 , calculate the density of each solution.
3- On your graph paper, draw a graph of concentration of solution vs. density of solution.
4- Calculate the density of soft drinks A and B and record your results in table 3.
5- Mark the density values you found for the soft drinks an your graph of concentration of solution vs. density of solution and determine the sugar concentration of the soft drinks.

## Part B: Determining the content of sugar with a laser: Index of refraction method.

Perform this part of the experiment in dim light to observe the laser light rays better.


Figure 3: Set up for index of refraction method.

1- Pour $10 \%$ solution in the hollow semicircular block.
2- Switch on the laser source and align the ray table such that incident and reflected ray angles are both on the normal (the dashed line in figure 2)
3- Turn the ray table slowly and observe the incident and refracted light rays. Bring the angle of incident ray to $40^{\circ}$. Measure the angle of refraction of the $10 \%$ sugar solution. Record your result in table 4.
4- Repeat your measurements 3 more times for the $10 \%$ sugar solution. Find the average of your results. Record your results in table 4.
5- Using a syringe, empty the hollow cylinder and fill it with plain water and empty it again.
6- Then repeat the procedure for $20 \%$ and then $30 \%$ sugar solutions. Record your results in table 4.
6- Now measure the angle of refraction of two soft drinks A and B with unknown sugar concentrations. Record your results in table 5 .

Table 4
Angle of incidence $=40^{\circ}, \operatorname{Sin} 40^{\circ}=0.6428$

| Concentration of solution (\%) | Refraction angle ( $\boldsymbol{\theta}_{2}$ ) |  |  |  |  | Sin of refraction angle $\left(\boldsymbol{\operatorname { s i n }} \boldsymbol{\theta}_{\mathbf{2}}\right)$ | Index of refraction of solution $\left(\mathbf{n}_{1}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Trial \#1 | Trial \#2 | Trial \#3 | Trial \#4 | Average |  |  |
| 10 |  |  |  |  |  |  |  |
| 20 |  |  |  |  |  |  |  |
| 30 |  |  |  |  |  |  |  |

Table 5

| Soft Drink | Refraction angle $\left(\boldsymbol{\theta}_{\mathbf{2}}\right)$ |  |  |  |  | Sin of <br> Refraction <br> angle $\left(\boldsymbol{\operatorname { s i n }} \boldsymbol{\theta}_{\mathbf{2}}\right)$ | Index of <br> refraction of <br> soft drink $\left(\mathbf{n}_{\mathbf{1}}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Trial \#1 | Trial \#2 | Trial \#3 | Trial \#4 | Average |  |  |
|  |  |  |  |  |  |  |  |
| B |  |  |  |  |  |  |  |

## Data Analysis for Part B

You will need a scientific calculator in this part!
1- Calculate the sin of refraction angle for each solution and record your results in table 4.

2- Using equation 3, calculate the index of refraction of each solution. Record your results in table 4.
3- On your graph paper, draw a graph of concentration of solution vs. index of refraction of solution.
4- Calculate the sin of refraction angle for soft drinks A and B and record your results in table 5.
5- Using equation 3, calculate the index of refraction of soft drinks. Record your results in table 5.
6- Mark the index of refraction values you found for the soft drinks on your graph of concentration of solution vs. index of refraction of solution and determine the sugar concentration of the soft drinks.

## Questions

1- Compare the two methods we used in this experiment. Which one was more accurate? Which one was more precise? Explain.
2- Do different wavelengths of light refract the same amount? Why would you use a laser in this experiment, instead of another source of light? Remember that a laser has exactly one wavelength of light.
3- Using your findings in this experiment, estimate the amount of sugar you take from soft drinks in a day.

## Reference

University Physics by H.D. Young and R.A. Freedman, vol. $213^{\text {th }}$ Edition, AddisonWesley (2012).

