Experiment #1: Measurement and Error Analysis

Objective

In this experiment we will get familiar with taking measurements and evaluate the experimental results. We are going to take length measurements with a tape measurer, time measurements with a stopwatch and perform error calculation on the derived quantity: gravitational acceleration. We will perform the experiment with a physical setting of a simple pendulum.

Theory

Simple Pendulum

A simple pendulum consists of a point-mass (*m*) suspended from a fixed point by a string of length (*L*). When the mass is displaced from vertical, it oscillates about the vertical plane. The relevant parameter that describes this oscillation is known as the period of oscillation. The period of oscillation (T) is the time required for the pendulum to complete one cycle in its motion. This can be determined by measuring the time required for the pendulum to reoccupy a given position.

The reason that the pendulum oscillates about the vertical is that if the pendulum is displaced, the force of gravity pulls down on the pendulum. The pendulum begins to move downward. When the pendulum reaches vertical it can't stop instantaneously. The pendulum continues past the vertical and upward in the opposite direction. The force of gravity slows it down until it eventually stops and begins to fall again. If there is no friction where the pendulum is fixed and there is no air resistance to the motion of the pendulum, this would continue forever.

Because it is the force of gravity that produces the oscillation, one might expect the period of oscillation to differ for differing values of gravity. In particular, if the force of gravity is small, there is less force pulling the pendulum downward, the pendulum moves more slowly toward vertical, and the observed period of oscillation becomes longer. Thus, by measuring the period of oscillation of a pendulum, we can estimate the gravitational force or acceleration.

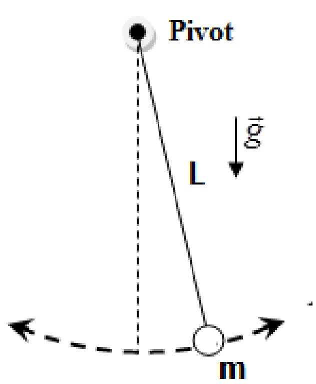

Figure 1: Simple Pendulum

As Show in the Figure, *m* is the mass of the pendulum bob, *g* is the magnitude of the gravitational acceleration and *L* is the length of the string. *m*, *g*, and *L* are positive constants. A real pendulum oscillating with a small amplitude may be described as a simple pendulum if the following two conditions are met:

1. The length of the string (L) is much greater than the size of the bob. In this case the bob can be treated as a point-mass located at the center of gravity of the bob.
2. The mass of the string supporting the mass is much less than the mass of the bob.

The period of the pendulum will be given by the following equation:

$$T = 2\pi\sqrt{L/g} \qquad (1)$$

Solving equation (4) for *g* yields;

$$g = 4\pi^2 L/T^2 \qquad (2)$$

The period of the pendulum can also be found experimentally by following equation:

$$T = t/n \qquad (3)$$

Where *n* is the number of cycles and *t* is the time taken for the number of cycles.

Units

Any number that is used to describe a physical phenomenon quantitatively is called physical quantity. When we measure a quantity we always compare it with some reference standard. When we say that the Mclaren P1 is 4.59 meters long, we mean that it is 4.59 times longer than a meter stick, which we define to be 1 meter long. Such standard defines a **unit** of the quantity.

To make reliable measurements, we need common standards. The system of units used by scientists around the world is called the International System, or SI.

There is a second system of units, the British System, which is used in the United Kingdom, Unided States and other a few countries, but mostly replaced by the SI units. When the fundamental units are defined, we can introduce larger and smaller units of the same physical quantities. The names of the additional units are derives by adding prefixes to the fundamental units. For example; Length of 1 kilometer (km) equals to 10^3 (1000) meter (m).

Commonly used units in SI system are listed in the last page.

Experimental Errors

Error means the difference between an experimental value and the true value. There are two types of experimental errors: random errors and systematic errors. The difference between the systematic and random errors can be seen by repeating a measurement of a physical quantity several times under the same conditions. **Random errors** are statistical fluctuations or variations in the measured data produced by the experimenter's inability to take the same measurement in exactly the same way to get exactly the same reading. Therefore, the readings will be spread about the true value as shown in Figure 2 (a). You will analyse the ramdom error in your measurements in this experiment. On the other hand, **systematic errors** are repeating inaccuracies that causes the measurements to constantly be either too high or too low. They are mostly due to defects in the measuring devices which make them continually present throughout the entire experiment. Therefore, the readings will always be displaced far from the true value as shown in Figure 2 (b). For that reason, systematic errors are difficult to detect and cannot be analyzed statistically.

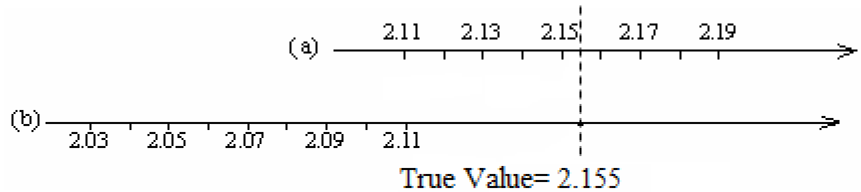

Figure 2:
(a) Measurements with only random errors.
(b) Measurements with both random and systematic errors.

• **The Accuracy** of an experiment is a measure of how close the result obtained for a given experiment is compared to the true value.
• **The Precision** of an experiment is a measure of how exactly the result is determined.
For example: The time is 09:20:331. One clock shows the time as 09:22:174, and the other clock shows the time as 09:20. So the first clock is more precise as it shows the time to the nearest 1 ms (miliseconds), and the second clock is more accurate as it show the time to the nearest 1s (seconds) but closer to the true value.

Uncertainty of Measuring Devices
A measure of precision of an instrument is given by its uncertainty. The uncertainty of a scale measuring device is the half of its smallest subdivision. For a tape measurer the smallest division is 1mm and half of that value represents the uncertainty which is ± 0.5 mm. For a digital measuring device, such as a stop watch, the smallest subdivision represents the uncertainty of any measurement.
For the clocks in our axample above; the first clock has uncertainty of ± 0.001 ms and the second clock has uncertainty of ± 1 s.

Significant Figures
The significant figures of a number are those digits that carry meaning contributing to its precision.
Rules for Determining Significant Figures in a Number are:
* All non-zero numbers are significant.
* Zeros within a number are always significant.

* Zeros that do nothing but set the decimal point are not significant. Both 0.000098 and 0.98 contain two significant figures.
* Zeros that aren't needed to hold the decimal point are significant. For example, 4.00 has three significant figures.
* Zeros that follow a number may be significant.
* When measurements are added or subtracted, the number of decimal places in the final answer should equal the smallest number of decimal places of any term.

Example adding two length measurements:

51.523 mm + 8.1 mm =59.623 mm

We round the result to the correct significant figure: 59.6 mm

* When measurements are multiplied or divided, the number of significant figures in the final answer should be the same as the term with the lowest number of significant figures.

Example mltiplying two length measurements:

2.2 cm * 33.45 cm = 73.59 cm^2

We round the result to the correct significant figure: 73 cm^2

Statistical analysis of random errors

Calculation of the Average Value

For a set of measurements, the equation below is used to calculate the average of the results:

$$x_{ave} = \frac{x_1 + x_2 + \ldots x_n}{n} \qquad (4)$$

Where x represents the physical parameter measured, x_{ave} is the average value of the measured parameter and n is the number of measurements.

Determining the Standard Deviation

To calculate the standard error of a measurement, we need to introduce the standard deviation (σ). Standard deviation measures the spread of the data about the average value. A low standard deviation indicates that the data points are tend to be very close to the average whereas high standard deviation means that the data points are spread out over a large range of values.

The standard deviation formula is:

$$\sigma = \sqrt{\frac{(x_1 - x_{ave})^2 + (x_2 - x_{ave})^2 + \ldots + (x_i - x_{ave})^2}{n-1}} \qquad (5)$$

Standard deviation is used to calculate the standard error ($\sigma_{x(ave)}$). The uncertainty or standart error in the average value is defined as:

$$\sigma_{x(ave)} = \frac{\sigma}{\sqrt{n}} \qquad (6)$$

The experimental result is expressed as:

$$x_{\exp} = x_{ave} \pm \sigma_{x(ave)}$$

Percent Error

Percent error is the percent difference of the experimental result with the true value. The equation for the percent error is:

$$\%Error = \left| \frac{Experimental\ \mathrm{Re}sult - True\,Value}{True\,Value} \right| \times 100 \qquad (7)$$

Safety

Use the lock on the tape measurer. Mind your finger when releasing it.
Pendulum bob is made of lead, a heavy metal. Never give it an oscillation with more than 5 °.
Report any damage of pendulum string to your lab instructor.
Make sure the clamp to the table and the connection to the stand rod is tight at all times.

Equipment list

*Measuring tape
**Stop watch*
**Pendulum*
**Clamp stand*

Procedure

The set up consists of a simple pendulum attached to a clamp stand.

1- Using tape measurer, measure the length of the pendulum. Record your result in the Table 1 with its uncertainty.
2- Start the bob oscillate through a small arc (about 5 ° on either side of vertical).Use a stop watch to measure the time it takes for the pendulum to complete 20 oscillations.
3- For the same length, repeat the experiment for 9 more times. Record the measured values in the data table 2.

Data Analysis

(Give attention to use correct units and significant figures.)

1- Use equation (3) and calculate the period of oscillation for each trial and record the results in the data table 1.
2- Use equation (2) and Calculate gravitational acceleration (g) for each trial and record the values in the data table 1.
3- Use equation (4) to calculate the average value of gravitational acceleration (g_{ave}) and record the values in the data table 2.
4- Use equation (5) and (6) to calculate the standard deviation and standard error of your result. And express you result as: $g_{\exp} = g_{ave} \pm \sigma_{ave}$ and record the values in the data table 2.
5- Use equation (7) to calculate the percent error to compare your experimental result with the true gravitational acceleration value in Istanbul ($g = 9.808 m/s^2$). Record the values in the data table 2.

Data Table 1

$L\,(m)$	

Trial #	*Time for 20 oscillations* $t\,(s)$	*Period* $T=t/20\,(s)$	*Gravitational Acceleration* $g\,(m/s^2)$
1			
2			
3			
4			
5			
6			

Data Table 2

Average Gravitational Acceleration	*(gave)* =	____________________
Standard Deviation	(σ) =	____________________
Standard Error of the Average Value	($\sigma_{x(ave)}$) =	____________________
Experimental Result	$x_{\exp} = x_{ave} \pm \sigma_{x(ave)}$ =	____________________

Questions

1- What are the types of errors you encountered in this experiment? Random or systematic? Explain. (Compare your gravitational acceleration results for each trial and the true gravitational acceleration value.)
2- What is the accuracy of your gravitational acceleration result? Explain.
3- Make three suggestions to increase the precision in your experiment.
4- If you were to double the mass on the end of the pendulum, what would happen to the period?
5- If you were to do this experiment on the moon, what would you expect of the motion of the pendulum?

Reference

University Physics by H.D. Young and R.A. Freedman, vol.2 13th Edition, Addison-Wesley (2012).

Appendix A

Metric Units (SI System)

Quantity	**SI unit and symbol**
Time	second (s)
Length	metre (m)
Mass	kilogram (kg)
Electric current	ampere (A)
Temperature	kelvin (K) degree Celsius (°C)
Area	square metre (m2)
Acceleration	(m•s^{-2})
Frequency	hertz (Hz)
Energy	joule (J)
Power	watt (W)
Force	newton (N)
Pressure	pascal (Pa)
Electric charge	coulomb (C)
Potential difference	volt (V)
Capacitance	farad (F)
Inductance	henry (H)
Electric resistance	ohm (Ω)
Magnetic flux density	tesla (T)
Magnetic field strength	(A/m)
Irradiance	(W/m2)
Illuminance	lux (lx)
[Radioactive] activity	becquerel (Bq)
Absorbed [radiation] dose	gray (Gy)

Metric prefixes			
Prefix	**Symbol**	**10^n**	**English word**
giga	G	10^9	billion
mega	M	10^6	million
kilo	k	10^3	thousand
hecto	h	10^2	hundred
deca	da	10^1	ten
		10^0	one
deci	d	10^{-1}	tenth
centi	c	10^{-2}	hundredth
milli	m	10^{-3}	thousandth
micro	μ	10^{-6}	millionth
nano	n	10^{-9}	billionth
pico	p	10^{12}	trillionth

Appendix B
Report format to be used in lab notebooks.

Title and # of the Experiment

Date:

Objective (10 pts.)
State your reason of performing this experiment. Write down your hypothesis, your prediction of the answer to the problem that will be investigated in the experiment.

Procedure (10 pts.)
List the instructions you followed during the experiment briefly.

Data Analysis (30 pts.)
Record your results and calculations in the experiment. This section includes tabulated numbers, graphs, and short explanations of observations. Graphs are to be drawn with a ruler, titled, and axis labeled.

Discussion (40 pts.)
Give answers to the questions asked in the manual. State the main results and your explanation of the results.

Conclusion (10 pts.)
State whether your hypothesis was correct or not and summarize what you have learned in the experiment.