Experiment #1: Measurement and Error Analysis

Objective

In this experiment we will get familiar with taking measurements and evaluate the experimental results. We are going to determine the length, diameter, and mass of cylindrical objects. Calculating of the density of the cylindrical objects and comparing with the accepted values of the density of the metals.

Equipment

Four cylindrical objects of different sizes made from the same material (aluminum).
• Measuring devices: Tape Measurer, Laboratory Balance.

Theory

Density ρ of any substance is defined as the mass *m* of a unit volume, V of that substance. The simple way of finding the density is by measuring the mass of a known volume of the substance and dividing this mass by the volume

$$\rho = \frac{m}{V} \qquad (1)$$

The volume of any solid object with a simple geometric form can be determined from its dimensions. Volume of a solid cylinder is;

$$V = \pi d^2 l \qquad (2)$$

Where d is the diameter and l is the length of the cylinder. π is a constant and can be taken as 3. If the body is not too small, can be conveniently measured by a tape measure. The mass of a solid object can be determined by the use of the laboratory balance.

Units

Any number that is used to describe a physical phenomenon quantitatively is called physical quantity. When we measure a quantity we always compare it with some reference standard. When we say that the McLaren P1 is 4.59 meters long, we mean that it is 4.59 times longer than a meter stick, which we define to be 1 meter long. Such standard defines a **unit** of the quantity. To make reliable measurements, we need common standards. The system of units used by scientists around the world is called the International System, or SI.
There is a second system of units, the British System, which is used in the United Kingdom, United States and other a few countries, but mostly replaced by the SI units. When the fundamental units are defined, we can introduce larger and smaller units of the same physical quantities. The names of the additional units are derives by adding prefixes to the fundamental units. For example; Length of 1 kilometer (km) equals to 10^3 (1000) meter (m).
Commonly used units in SI system are listed in the last page.

Experimental Errors

Error means the difference between an experimental value and the true value. There are two types of experimental errors: random errors and systematic errors. The difference between the systematic and random errors can be seen by repeating a measurement of a physical quantity several times under the same conditions. **Random errors** are statistical fluctuations or variations in the measured data produced by the experimenter's inability to take the same measurement in exactly the same way to get exactly the same reading. Therefore, the readings will be spread about the true value as shown in Figure 2 (a). You will analyze the random error in your measurements in this experiment. On the other hand, **systematic errors** are repeating inaccuracies that cause the measurements to constantly be either too high or too low. They are mostly due to defects in the measuring devices which make them continually present throughout the entire experiment. Therefore, the readings will always be displaced far from the true value as shown in Figure 2 (b). For that reason, systematic errors are difficult to detect and cannot be analyzed statistically.

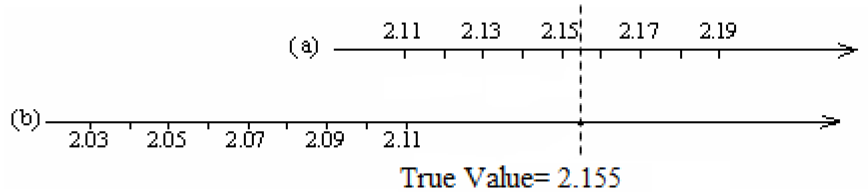

Figure 2:
(a) Measurements with only random errors.
(b) Measurements with both random and systematic errors.

• **The Accuracy** of an experiment is a measure of how close the result obtained for a given experiment is compared to the true value.
• **The Precision** of an experiment is a measure of how exactly the result is determined.
For example: The time is 09:20:331. One clock shows the time as 09:22:174, and the other clock shows the time as 09:20. So the first clock is more precise as it shows the time to the nearest 1 ms (milliseconds), and the second clock is more accurate as it show the time to the nearest 1s (seconds) but closer to the true value.

Uncertainty of Measuring Devices

A measure of precision of an instrument is given by its uncertainty. The uncertainty of a scale measuring device is the half of its smallest subdivision. For a tape measure the smallest division is 1mm and half of that value represents the uncertainty which is ± 0.5 mm. For a digital measuring device, such as a laboratory balance, the smallest subdivision represents the uncertainty of any measurement.
For the clocks in our example above; the first clock has uncertainty of ± 0.001 ms and the second clock has uncertainty of ± 1 s.

Statistical analysis of random errors

Calculation of the Average Value

For a set of measurements, the equation below is used to calculate the average of the results:

$$x_{ave} = \frac{x_1 + x_2 + \dots x_n}{n} \qquad (3)$$

Where x represents the physical parameter measured, x_{ave} is the average value of the measured parameter and n is the number of measurements.

Determining the Standard Deviation

To calculate the standard error of a measurement, we need to introduce the standard deviation (σ). Standard deviation measures the spread of the data about the average value. A low standard deviation indicates that the data points are tend to be very close to the average whereas high standard deviation means that the data points are spread out over a large range of values. The standard deviation formula is:

$$\sigma = \sqrt{\frac{(x_1 - x_{ave})^2 + (x_2 - x_{ave})^2 + \dots + (x_i - x_{ave})^2}{n-1}} \qquad (4)$$

Percent Error

Percent error is the percent difference of the experimental result with the true value. The equation for the percent error is:

$$\%Error = \left| \frac{Experimental\ Result - True\ Value}{True\ Value} \right| \times 100 \qquad (5)$$

Safety

1- Use the lock on the tape measure. Mind your finger when releasing it.
2- Keep Aluminum cylinders in container to avoid dropping them to your feet.
3- Hold the insulating part of the socket when disconnecting the plug of Laboratory balance.
4- Report any damage to the apparatus to your lab instructor.

Procedure

1) Use the tape measure to measure the diameter (d) of the cylindrical objects. Record your data in Table 1.
2) Use the laboratory balance to measure the mass (m) of the cylindrical objects. Record your data in Table 1.

Table 1

Object #	*Diameter d (cm)*	*Length l (cm)*	*Mass m (g)*	*Volume* $V(cm^3)$	*Density* $\rho\ (\frac{g}{cm^3})$
Cylinder 1					
Cylinder 2					
Cylinder 3					
Cylinder 4					

Data Analysis

1) Calculate the volume for each cylindrical object using equation 2.
2) Calculate the density (ρ) for each piece using equation 1.
3) Using equation 3, compute the average value of the density $(\rho_{ave.})$ and record the value in table 2.
4) Using equation 4, calculate the standard error (σ) of your results.
5) Use your data on table 1 to plot a graph of the mass m (y-axis) of each cylindrical object versus its volume V (x-axis). According to equation 1, the graph should be a straight line through the origin. Get the average density ($\rho_{ave.}$) from the slope of this line and record the value in table 2.
6) Using equation 5, compare the measured density with the accepted value for the density of aluminum metal.

Table 2

Average density from table 1	$(\rho_{ave.})$ = ____________________
Standard deviation	(σ) = ____________________
Average density from the graph	$(\rho_{ave.})$ = ____________________

Questions

1- What are the types of errors you encountered in this experiment? Random or systematic? Explain. (Compare your average density results for each trial and the accepted density value.)
2- Experimental results within 5% of the true value fall in the accepted range. What is the accuracy of your density result? Explain.
3- Make two suggestions to increase the precision in your experiment.

Reference

University Physics by H.D. Young and R.A. Freedman, vol.2 13th Edition, Addison-Wesley (2012).

Appendix

Metric Units (SI System)

Quantity	**SI unit and symbol**
Time	second (s)
Length Mass	meter (m) kilogram (kg)
Electric current	ampere (A)
Temperature	kelvin (K) degree Celsius (°C)
Area	square meter (m2)
Acceleration	($m \cdot s^{-2}$)
Frequency	hertz (Hz)
Energy	joule (J)
Power	watt (W)
Force	newton (N)
Pressure	pascal (Pa)
Electric charge	coulomb (C)
Potential difference	volt (V)
Capacitance	farad (F)
Inductance	henry (H)
Electric resistance Magnetic flux density	ohm (Ω) tesla (T)
Magnetic field strength	(A/m)
Irradiance	(W/m2)
Illuminance	lux (lx)
[Radioactive] activity	becquerel (Bq)
Absorbed [radiation] dose	gray (Gy)

Metric prefixes			
Prefix	**Symbol**	**10^n**	**English word**
giga	G	10^9	billion
mega	M	10^6	million
kilo	k	10^3	thousand
hecto	h	10^2	hundred
deca	da	10^1	ten
		10^0	one
deci	d	10^{-1}	tenth
centi	c	10^{-2}	hundredth
milli	m	10^{-3}	thousandth
micro	µ	10^{-6}	millionth
nano	n	10^{-9}	billionth